

# Melville Senior High School

### Semester Two Examination, 2019

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**METHODS**

**UNITS 1 AND 2**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working  time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

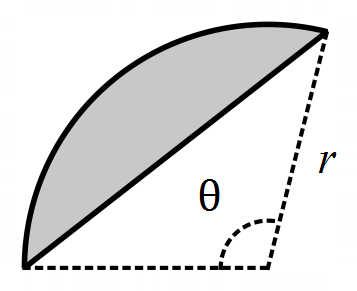
Working time: 100 minutes.

Question 9 (6 marks)

(a) Convert to an exact radian measure. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(b) A segment of a circle of radius cm is shown below, where .



(i) Determine the area of the segment. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates correct use of formula   correct area |

(ii) Determine the perimeter of the segment. (3 marks)

|  |
| --- |
| **Solution** |
| Arc length is and chord length is . |
| **Specific behaviours** |
| ✓ arc length   use of cosine rule for chord length   correct perimeter |

Question 10 (8 marks)

From a random survey of telephone usage in households it was found that households had access to a mobile phone but not a landline, households had access to a landline and more households had access to a mobile phone than did not.

(a) Complete the missing entries in the table below. (3 marks)

|  |  |  |  |
| --- | --- | --- | --- |
|  | Mobile | No mobile | Total |
| Landline |  |  |  |
| No landline |  |  |  |
| Total |  |  |  |

|  |
| --- |
| **Solution** |
| See table |
| **Specific behaviours** |
| ✓ totals column;  totals row;  rest of table |

(b) If one household is randomly selected from those surveyed, determine the probability that

(i) it had access to a landline. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) it had no access to a mobile phone given that it had access to a landline. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(iii) it had access to a landline given that it no access to a mobile phone. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(c) Use your answers above to comment on the possible independence of households having access to a mobile phone and households having access to a landline. (2 marks)

|  |
| --- |
| **Solution** |
| Strong indication that the events are independent as - would expect these probabilities to be further apart if not independent. |
| **Specific behaviours** |
| ✓ states independent   justifies by comparing probabilities |

Question 11 (7 marks)

A drone is flying in a straight line and at a constant height m above a level pitch towards a thin goal post. It maintains a constant speed of ms-1.

Initially, the angle of depression from the drone to the base of the post is . Exactly seconds later this angle has increased to .

(a) Sketch a diagram to show the two angles of depression from the drone to the base of the post. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ sketch (measurements not required) |

(b) Determine, showing all working, the value of and calculate the time after leaving its initial position that the drone will collide with the post. (6 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates distance travelled   writes equation using trig   writes second equation using trig   solves equations   states   states time |

Question 12 (5 marks)

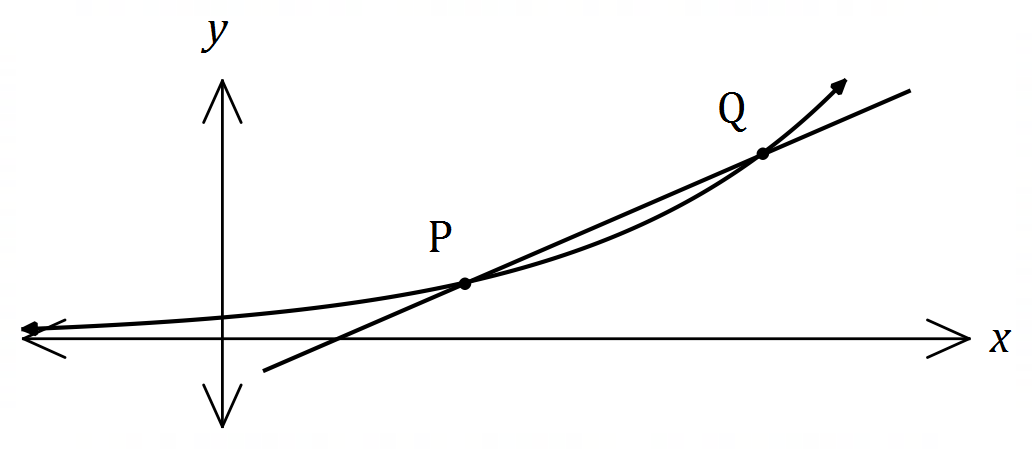
A geometric sequence has a second term of and a sum to infinity of .

Determine the sum of the first terms of the sequence.

|  |
| --- |
| **Solution** |
| Solving simultaneously gives  (ignore since for sum to infinity) |
| **Specific behaviours** |
| ✓ equation using   equation using   solves for and   discards invalid solution   calculates |

Question 13 (7 marks)

The graph of is shown below, where , together with the secant to the curve through the points and .



has coordinates and has coordinates where .

(a) Complete the second column in the table below, rounding values to decimal places where necessary. (4 marks)

|  |
| --- |
| **Solution** |
| See table |
| **Specific behaviours** |
| ✓ one correct value   three correct values   all correct   last all to dp |

|  |  |
| --- | --- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

(b) Name the feature of the graph above that the values you calculated in part (a) represent.

(1 mark)

|  |
| --- |
| **Solution** |
| Values are gradient of secant as moves closer to . |
| **Specific behaviours** |
| ✓ indicates gradient of secant |

(c) Determine an estimate, correct to decimal places, for the value that approaches as becomes closer and closer to and state what this value represents.

(2 marks)

|  |
| --- |
| **Solution** |
| Value approaches .  Value is gradient of curve at . |
| **Specific behaviours** |
|  correct value   states value approaches gradient at point |

Question 14 (10 marks)

When a manufacturer makes litres of a chemical using process , the cost in dollars per litre varies according to the rule

(a) Determine

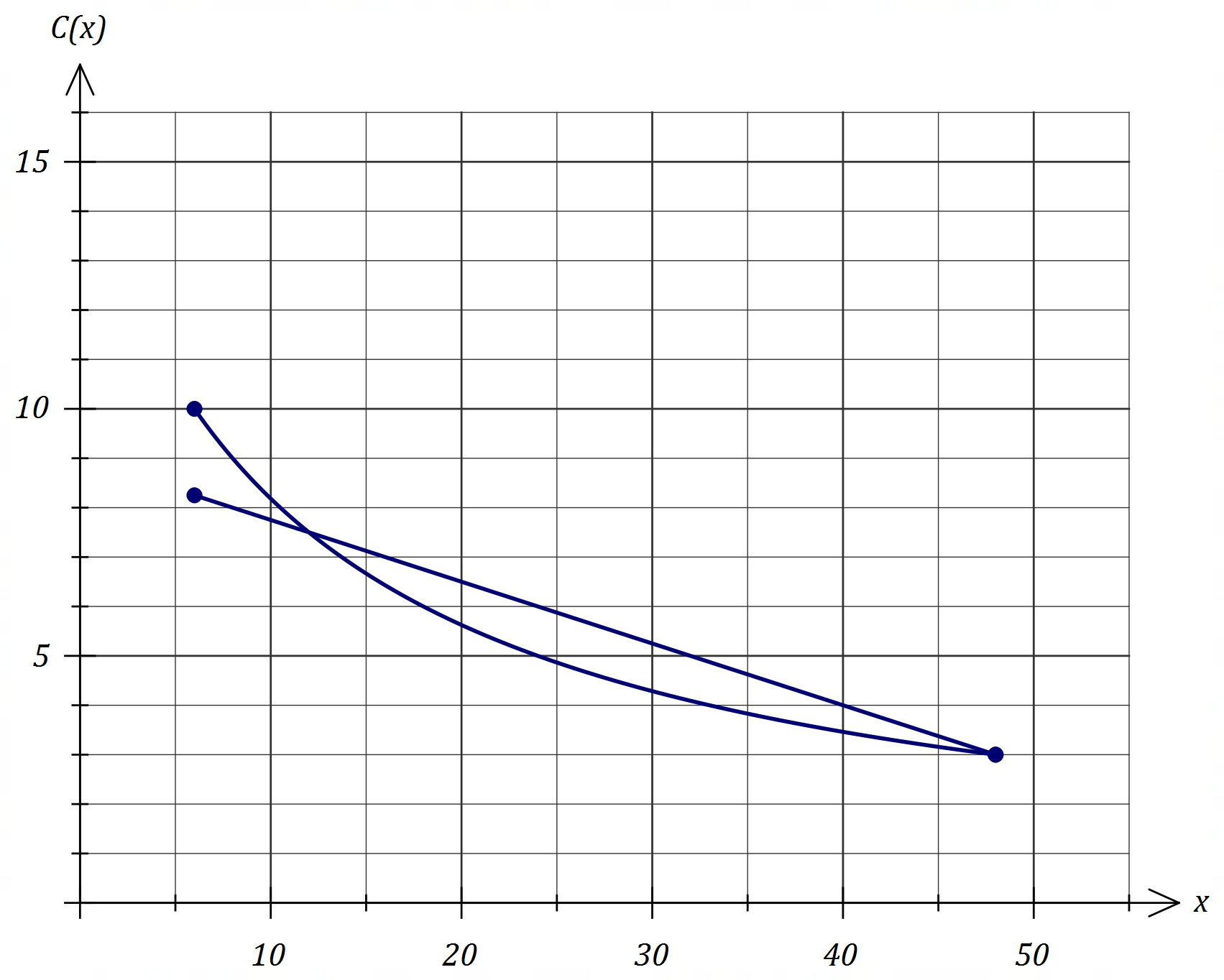
(i) the cost per litre when L is made. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct cost per litre |

(ii) the total cost of making L of the chemical. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ cost per litre   total cost |

(b) Graph the cost per litre over the given domain on the axes below. (3 marks)



|  |
| --- |
| **Solution (b)** |
| See graph |
| **Specific behaviours** |
| ✓ LH endpoint   RH endpoint   smooth curve close to and |

(c) State the range of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct range |

When the manufacturer uses process , the cost in dollars per litre is modelled by

(d) Add this function to the graph and hence determine the production quantities for which process is cheaper than process . (3 marks)

|  |
| --- |
| **Solution** |
| See graph for line.  Process is cheaper than for litres. |
| **Specific behaviours** |
| ✓ ruled line through and   correct bounds   does not include bounds in answer |

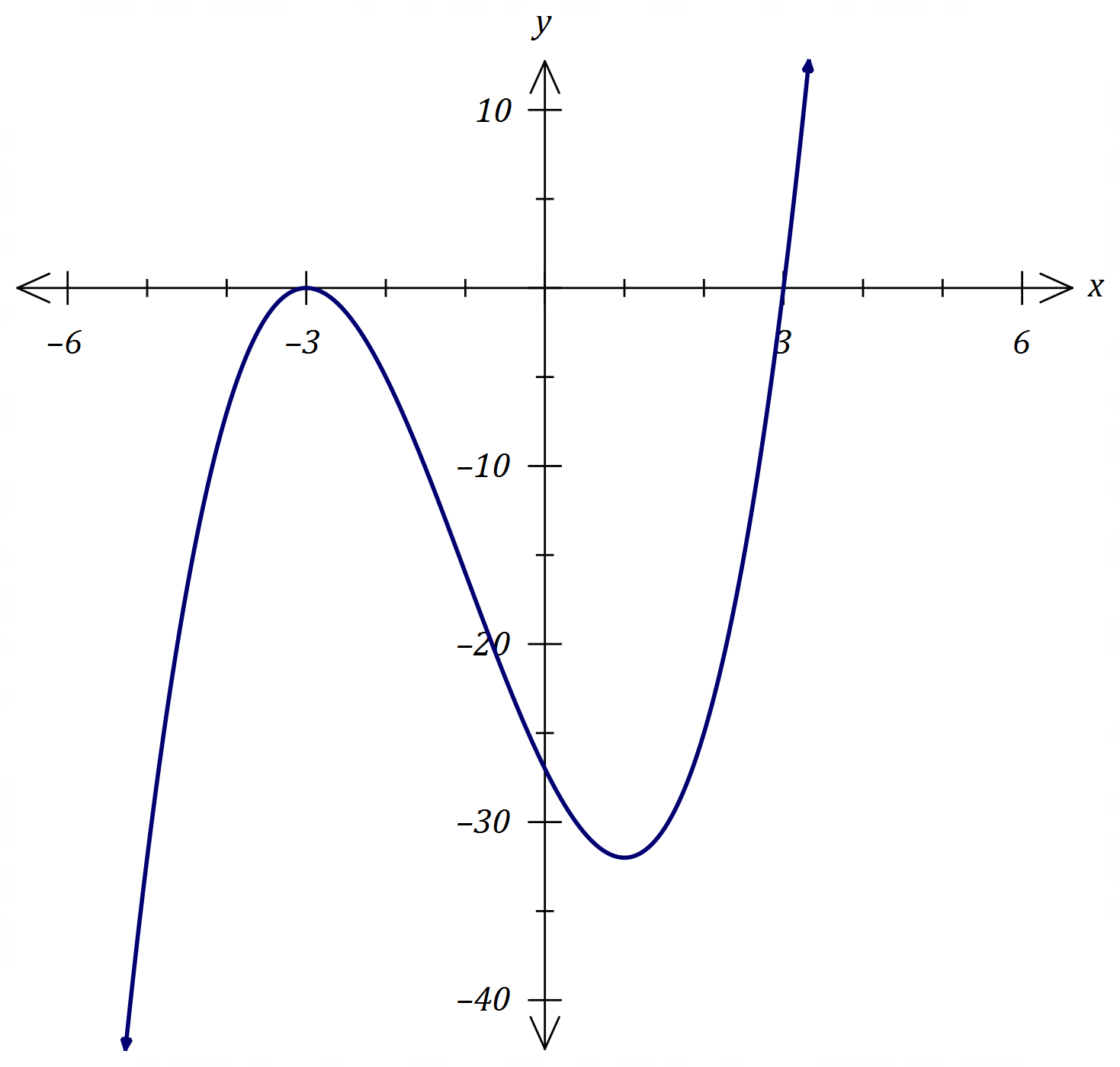
Question 15 (12 marks)

The function is defined by , where and are constants.

The graph of has the following features:

* passes through and
* has a local maximum at

(a) Sketch the graph of on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ local maximum   y-intercept and root   smooth curve |

(b) Determine the value of , the value of and the value of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ writes in factored form   expands   states all three values |

(c) Use a calculus method to determine the exact coordinates of the local minimum of the graph of . (3 marks)

|  |
| --- |
| **Solution** |
| Local minimum at |
| **Specific behaviours** |
| ✓ shows   shows and solutions   correct coordinates |

(d) Determine the coordinates of the point where the tangent to at intersects the curve , other than at the point of tangency. (3 marks)

|  |
| --- |
| **Solution** |
| Tangent:  Intersects at |
| **Specific behaviours** |
| ✓ equation of tangent   equates tangent to curve and solves   correct coordinates |

Question 16 (7 marks)

When a patient takes a painkilling drug , the probability that they experience some side effects is known to be .

(a) A doctor prescribes drug to two unrelated patients. Determine the probability that

(i) neither patient experiences some side effects. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(ii) one patient experiences some side effects and the other does not. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates   doubles to obtain correct probability |

Other painkilling drugs are available. Of those who take drug , of patients who suffer some side effects will switch to another drug whereas no patient who has no side effects will switch.

(b) The doctor prescribes drug to a patient. Determine the probability that the patient does not switch to another drug. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ probability of side effect and does not switch   correct probability |

(c) The doctor prescribes drug to three unrelated patients. Determine the probability that at least one of these patients switch to another drug. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ probability none switch   correct probability |

Question 17 (7 marks)

The amount of water in a tank, litres, varies with time , in minutes, and can be modelled by the equation , .

(a) Determine amount of water in the tank

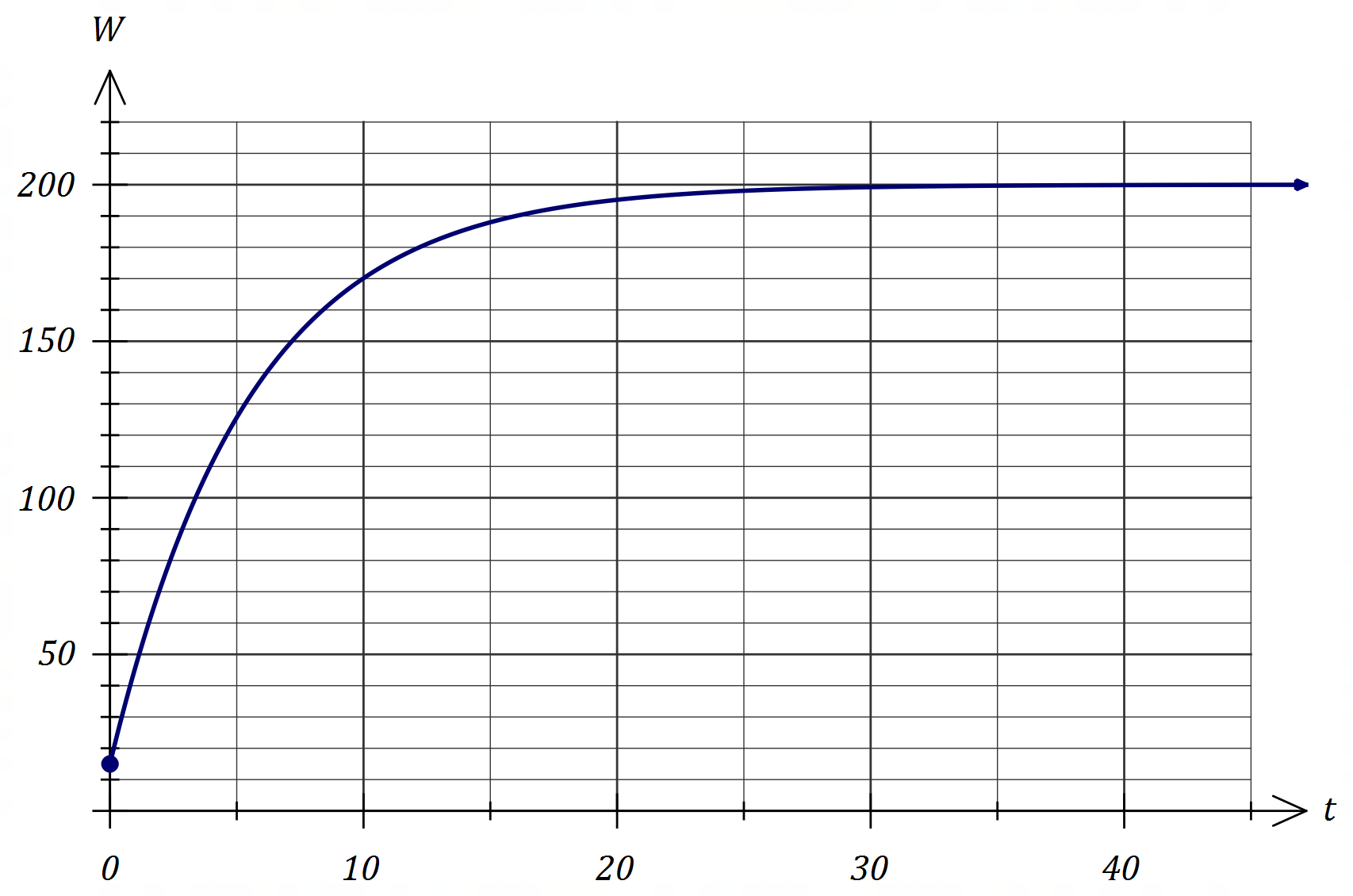
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(i) initially. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) after minutes. (1 mark)

(b) Graph against for on the axes below. (3 marks)



|  |
| --- |
| **Solution** |
| See graph |
| **Specific behaviours** |
| ✓ -intercept and   asymptotic behaviour   smooth curve |

(c) Over time, the amount of water in the tank approaches litres. State the value of and determine the time at which the amount of water in the tank reaches of this value.

(2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value of   correct time |

Question 18 (8 marks)

Two events and are such that and .

(a) Determine when . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ use of Venn diagram or other method   correct probability |

(b) Determine an expression for in terms of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(c) Determine the value of when

(i) and are mutually exclusive. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct value |

(ii) and are independent. (2 marks)

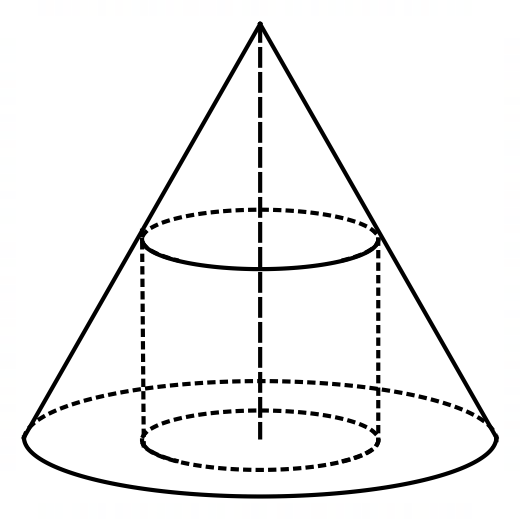
|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses rule for independence   correct value |

(iii) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses conditional probability rule   correct value |

Question 19 (7 marks)

A right circular cone of base radius cm and height cm stands on a horizontal surface. A cylinder of radius cm and volume cm3 stands inside the cone with its axis coincident with that of the cone and such that the cylinder touches the curved surface of the cone as shown.



(a) Show that . (3 marks)

|  |
| --- |
| **Solution** |
| From similar triangles  Hence |
| **Specific behaviours** |
|  relation between and using similar triangles   expresses in terms of   substitutes into cylinder volume formula |

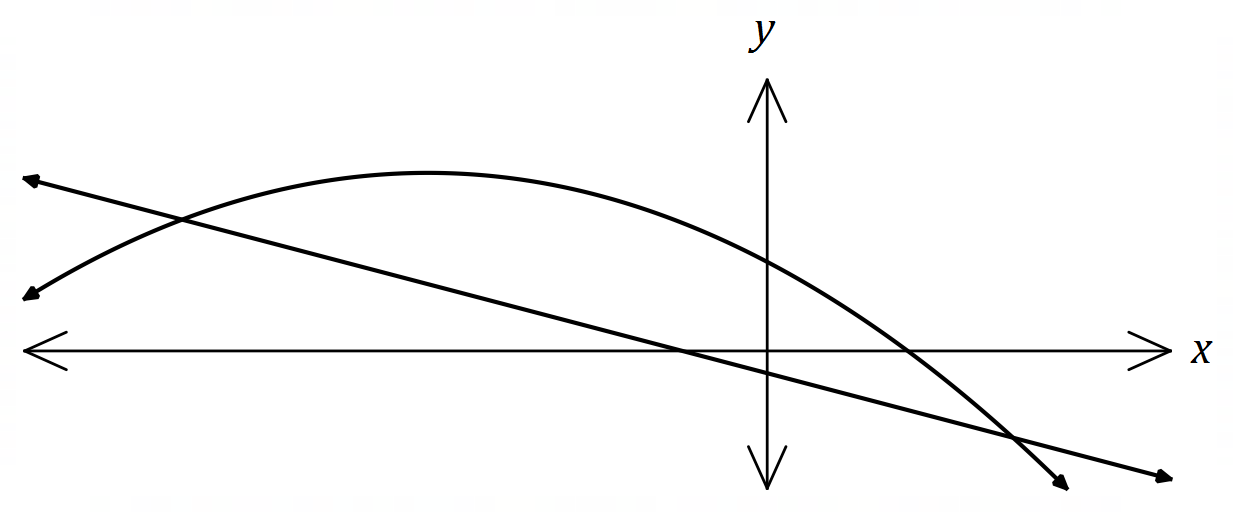
(b) Given that can vary, use a calculus method to determine the maximum value of .

(4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ derivative   equates derivative to   solves for   states maximum volume |

Question 20 (6 marks)

The graphs of and are shown below where and  
.



Determine the value(s) of the constant so that the equation has

(a) one solution. (5 marks)

|  |
| --- |
| **Solution** |
| must be a tangent to :  -coordinate of point of tangency:  Equation of tangent:  Hence |
| **Specific behaviours** |
| ✓ indicates tangent required   determines -coordinate of point of tangency   determines -coordinate of point of tangency   equation of tangent   states correct value of |

(b) two solutions. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct inequality |

Question 21 (8 marks)

A fair eight-sided dice numbered and is thrown times until it lands on an .

(a) Show that the probability that is . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ shows product of three fractions |

(b) Determine the probability that . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct probability |

(c) Write an expression in terms of for the probability that the first is thrown on the th throw and explain why the probabilities form a geometric sequence. (2 marks)

|  |
| --- |
| **Solution** |
| The expression takes the form of the th term of a GP: |
| **Specific behaviours** |
| ✓ correct expression   compares to general term of GP |

(d) Determine the probability that the first is thrown in or less attempts. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates use of sum formula   correct probability |

(e) The probability that the first is thrown in or less attempts must be at least . Determine the least value of integer . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ solves for   correct value of |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

© 2019 WA Exam Papers. Melville Senior High School has a non-exclusive licence to copy and communicate this document for non-commercial, educational use within the school. No other copying, communication or use is permitted without the express written permission of WA Exam Papers. SN063-142-4.